Shear response of a smectic film stabilized by an external field

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The response of a field-stabilized two-dimensional smectic liquid crystal to shear stress is discussed. Below a critical temperature the smectic film exhibits elastic response to an infinitesimal shear stress normal to the layering. At finite stresses free dislocations nucleate and relax the applied stress. The coupling of the dislocation current to the stress results in non-Newtonian viscous flow. The flow profile in a channel geometry is shown to change qualitatively from a power-law dependence to a Poiseuille-like profile upon increasing the pressure head.

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I. INTRODUCTION

Thin layers of elongated molecules exhibit a variety of different liquid crystalline phases differing in the degree of positional and orientational order from isotropic films. Complex rheological properties are common, in particular, when external orienting fields affect the alignment of the molecules. The coupling of the velocity field to the liquid-crystalline order gives rise to anisotropic viscosities. Furthermore, nonlinearities introduce shear-rate-dependent effective viscosities, which include phenomena like shear thinning, etc.

Recent experiments on Langmuir monolayers [1–3] on an air-water interface demonstrate the flow-induced microstructural deformations and orientation within a liquid-crystalline film. In particular, the nonlinear shear response gives rise to unconventional flow profiles in channel geometry. Since dislocations often play a prominent role in two-dimensional systems, the motion of these point defects is presumably responsible for many of the observed phenomena.

Here we focus on a smectic film floating on a liquid substrate. Such systems can be realized by suspending certain disk-shaped molecules on an air-water interface [4–8]. These disk-shaped molecules exhibit columnar phases in three dimensions. Restricted to two-dimensional interfaces these systems exhibit two phases reminiscent of the three-dimensional columnar order. For the face-on configuration the structure factor reveals hexatic order, i.e., short-range positional order and quasi-long-range orientational order for the nearest-neighbor bond vector. In the second case, the edge-on configuration, the molecules order in two-dimensional columns with liquidlike positional order within the columns. This second case is thus the two-dimensional analog of a smectic phase, which we discuss in this paper.

Smectic films are two-dimensional layered systems characterized by an elastic free energy that descibes long-wavelength distortions of the layers in terms of a displacement field $u(\vec{r})$ [9]. Up to quadratic order, we have

$$\mathcal{F}=\frac{1}{2}\int d^2r[B(\partial_z u)^2+B\lambda^2(\partial_x^2 u)^2], \qquad (1)$$

where the first term deals with layer compression, while the second term is related to splay distortions. The thermal fluctuations destroy even quasi-long-range translational order at any nonzero temperature. One easily finds that phonons give rise to exponential decay in the order parameter $\psi(\vec{r})=\exp[iq_0u(\vec{r})]$, where $q_0=2\pi/d$ is the wave vector associated with the layer spacing d [10]. In order to moderate the effect of thermal fluctuations one can apply an in-plane orienting external field. The corresponding free energy then has to be modified to [11]

$$\mathcal{F}=\frac{1}{2}\int d^2r[B(\partial_z u)^2+E(\partial_x u)^2]. \qquad (2)$$

Here E measures the coupling to the external field, e.g., an electric or magnetic field. The nematic director orientation is aligned with the displacement field according to $\theta=-\text{const}\,\partial_x u$ on large scales. This property does not necessarily hold near the boundaries. As discussed in the Appendix, there appears a characteristic length scale on which the director adjusts to the displacement.

One can eliminate the anisotropy in Eq. (2) by a simple volume-preserving rescaling, and the static properties are thus determined by the universality class of the XY model. There is a critical temperature $T_c\propto\sqrt{BE}$ separating a low-temperature smectic phase characterized by bound dislocations and power-law correlations in $\langle\psi(\vec{r})^*\psi(0)\rangle$ and a high-temperature nematic phase with exponential decaying order parameter correlations and free dislocations. Varying the external field allows control of the anisotropy. In particular, since the ratio E/B is preserved under renormalization, one can adjust experimentally the critical temperature T_c.

Here we discuss the dynamical properties of the stabilized smectic liquid crystal below T_c. The long-wavelength renormalized stiffness constants E_R and B_R now attain finite nonzero values and the film reacts as an elastic medium to an infinitesimal applied shear stress perpendicular to the layers. However, a finite stress can trigger the proliferation of free dislocations that result in shear flow.

II. HYDRODYNAMICS

The long-wavelength and low-frequency dynamics of a smectic film is governed by hydrodynamic equations for the broken symmetry variable, viz., the layer displacement u, and the conserved quantities [9]. For good thermal conductivity on a water substrate no temperature gradients can build up and energy conservation can be ignored. Furthermore we specialize to incompressible smectic films, setting the mass density $\rho=$const. Mass conservation then implies the constraint $\partial_x g_x+\partial_z g_z=0$ for the momentum density $\vec{g}(x,z)$. Similarly the trace of the viscous stress tensor also drops out of this long-wavelength low-frequency description. The viscous stress tensor then has only two independent components that have to be related to the two of the strain rate tensor. From symmetry there appear two indpendent viscosities and we write the constitutive equation for the viscous stress as $\sigma'_{xz}=\nu(\partial_z g_x+\partial_x g_z)$, $\sigma'_{xx}=(\nu-\nu')\partial_x g_x$, $\sigma'_{zz}=(\nu-\nu')\partial_z g_z$. The linearized equations of motion then read

$$\partial_t u = g_z/\rho + \lambda_p(B\partial_z^2 + E\partial_x^2)u, \tag{3a}$$

$$\partial_t g_x = -\partial_x p + \nu\partial_z^2 g_x + \nu'\partial_x\partial_z g_z, \tag{3b}$$

$$\partial_t g_z = -\partial_z p + (B\partial_z^2 + E\partial_x^2)u + \nu\partial_x^2 g_z + \nu'\partial_z\partial_x g_x, \tag{3c}$$

where λ_p denotes the permeation constant and p the pressure.

If a constant external shear stress σ_{xz}^{ext} is applied, the medium reacts by building up a stationary viscous momentum flow $\sigma_{xz}^{ext}=\nu\partial_z g_x$. Since the layers are not distorted no elastic stresses can build up to balance the external stress. However, the medium does sustain an infinitesimal external stress σ_{zx}^{ext} by straining layers according to $\sigma_{zx}^{ext}=E\partial_x u$.

The preceding paragraphs correctly describe the linear response of the stabilized smectic film. The nonlinear response, however, is qualitatively different. Finite applied stresses create free dislocations in the displacement field that can move and relax the stress. For two-dimensional smectics dislocations have Burgers' vector $\pm d$ along the z axis (see Fig. 1). Hence, we can think of these dislocations as *scalar* defects with charge $m_\nu=\pm 1$. In the presence of plastic flow due to the dislocation motion the film now exhibits viscous behavior. A convenient way to incorporate the effects of dislocations is by switching from a description in terms of the displacement field u to the strains s_x, s_z. This avoids the introduction of branch cuts in the displacement field $u(x,z)$. Locally the strains are given by $s_x=\partial_x u, s_z=\partial_z u$. However, since the displacement field is no longer a single-valued function, the line integral $\oint_\Gamma(s_x dx+s_z dz)$ does not vanish for closed loops Γ, but rather counts the number of enclosed dislocations in units of the layer spacing d. Consequently the curl of the strain is given by $\partial_x s_z-\partial_z s_x=m(\vec{r})d$, where $m(\vec{r})=\Sigma_{\nu=\pm 1}m_\nu n_\nu(\vec{r})$ is the total ''charge'' dislocation density. The charges $m_\nu=\pm 1$ characterize the point dislocations and $n_\nu(\vec{r})=\Sigma_{i_\nu}\delta(\vec{r}-\vec{r}_{i_\nu})$ is the number density of dislocations of charge m_ν. Since dislocations can be created only in pairs, the overall dislocation density is conserved,

$$\partial_t m + \mathrm{div}\,\vec{J} = 0. \tag{4}$$

Here $\vec{J}$ denotes the two-dimensional current of dislocation charge. In terms of the strains, Eq. (4), is merely the integrability condition that allows us to set $\partial_t s_z + J_x d = \partial_z \Xi$, $\partial_t s_x - J_z d = \partial_x \Xi$. Choosing $\Xi = g_z/\rho + \lambda_p(B\partial_z s_z + E\partial_x s_x)$ ensures compatability with the dislocation-free dynamics, Eq. (3a). Upon collecting terms one derives

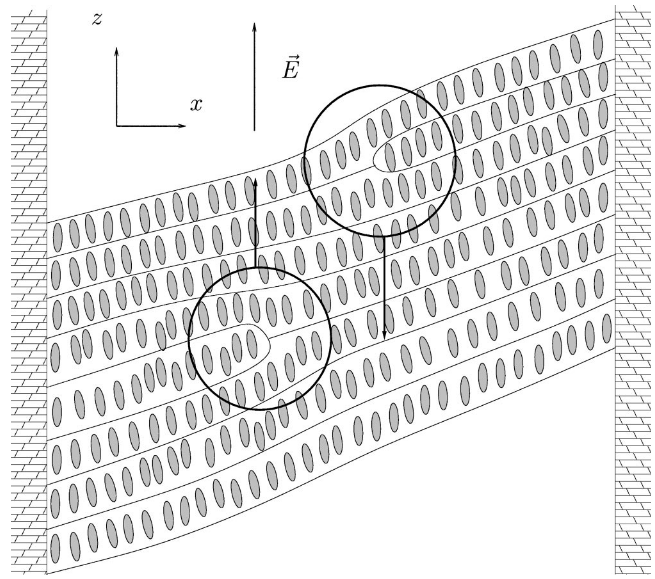

FIG. 1. Layering of the elongated molecules of a two-dimensional smectic. The dislocation on the right half has positive ''charge,'' whereas the dislocation on the left possesses negative ''charge,'' The strain s_x exerts a Peach-Koehler force on the free dislocations indicated by the arrows. The induced dislocation current then relaxes the strain.

$$\partial_t s_x = \partial_x g_z/\rho + \lambda_p\partial_x(B\partial_z s_z + E\partial_x s_x) + J_z d, \tag{5a}$$

$$\partial_t s_z = \partial_z g_z/\rho + \lambda_p\partial_z(B\partial_z s_z + E\partial_x s_x) - J_x d, \tag{5b}$$

$$\partial_t g_x = -\partial_x p + \nu\partial_z^2 g_x + \nu'\partial_x\partial_z g_z, \tag{5c}$$

$$\partial_t g_z = -\partial_z p + (B\partial_z s_z + E\partial_x s_x) + \nu\partial_x^2 g_z + \nu'\partial_z\partial_x g_x. \tag{5d}$$

For the dislocation current we adopt a Fokker-Planck description. There is a diffusive current due to gradients in the dislocation density. Symmetry dictates that the principal axis of the diffusion tensor are aligned with the (x,z)-coordinate system of the layers. Furthermore stresses induce motions that result in a separation of dislocations of opposite charge. Following Ref. [10] we write

$$J_x = \Gamma_x B d s_z n - \Gamma_x k_B T \partial_x m, \tag{6a}$$

$$J_z = -\Gamma_z E d s_x n - \Gamma_z k_B T \partial_z m. \tag{6b}$$

The first terms on the right-hand side are known as the Peach-Koehler forces and are the analog of the Magnus force in a superconductor, see Fig. 1. The diffusion constants $D_x=\Gamma_x k_B T$ and $D_z=\Gamma_z k_B T$ refer to dislocation climb and glide. Since climb, i.e., motion perpendicular to the Burgers vector, involves long-range mass transport one expects $\Gamma_x \ll \Gamma_z$. The free dislocation density $n(\vec{r})=\Sigma_\nu m_\nu^2 n_\nu(\vec{r})$ vanishes in the equilibrium smectic phase since all dislocations are bound. Thus the *linear* dynamics given by Eqs. (5) and (6) merely adds an anisotropic diffusive mode for dislocation motion to Eqs. (3).

A nonzero free dislocation density $n(\vec{r})$ leads to viscous response to an external shear σ_{zx}^{ext} as shown in Ref. [13]. Shearing perpendicular to the layers of the smectic film to produce a nonzero $s_x \approx \partial_x u$ liberates free dislocations that then can relax the shear. Thus one has a most prominent form of the phenomenon known as shear thinning, i.e., a reduction from an infinite linear viscosity and a nonzero linear elastic modulus to a finite nonlinear viscosity and zero nonlinear elastic shear modulus. This is the subject of the next two sections where we adopt methods applied to superfluid helium films in Ref. [12] to smectic hydrodynamics.

III. NUCLEATION RATE OF FREE DISLOCATIONS

The free energy of Eq. (2) contains contributions from the smooth part of the strain and from the interaction energy of dislocations. The strains can be decomposed as $s_x = \partial_x \phi + s_x^{sing}$, $s_z = \partial_z \phi + s_z^{sing}$. Then the smooth part and the dislocation interaction decouple provided one imposes the stress equlibrium condition

$$E \partial_x s_x^{sing} + B \partial_z s_z^{sing} = 0. \tag{7}$$

This relation is in fact the integrability condition that allows the introduction of the Airy stress function $E s_x^{sing} = -\partial_z \chi$, $B s_z^{sing} = \partial_x \chi$. Since the curl of the strain yields the dislocation density, the Airy stress function is given in terms of the solution of

$$\frac{1}{B} \partial_x^2 \chi + \frac{1}{E} \partial_z^2 \chi = m(\vec{r}) d. \tag{8}$$

After a partial integration the free energy Eq. (2) can be written

$$\mathcal{F} = \mathcal{F}_0 - \tfrac{1}{2} \int d^2 r \chi(\vec{r}) m(\vec{r}) d, \tag{9}$$

where the smooth fluctuations of the displacement field are encoded in

$$\mathcal{F}_0 = \tfrac{1}{2} \int d^2 r [B(\partial_z \phi)^2 + E(\partial_x \phi)^2]. \tag{10}$$

The dislocation contribution is the smectic analog of the interaction energy of charges in electrostatics. Here, $m(\vec{r})$ corresponds to the electric charge density and $\chi(\vec{r})$ to the electrostatic potential. As mentioned in Sec. I the electrostatic analogy becomes complete after the volume preserving transformation $x = \alpha \xi$, $z = \zeta / \alpha$, with $\alpha = (E/B)^{1/4}$. Equation (8) then reads

$$\partial_\xi^2 \chi + \partial_\zeta^2 \chi = m d B \alpha^2. \tag{11}$$

The bare interaction energy of a pair of dislocations of opposite charge ± 1 is then easily calculated to be

$$U_0(\varrho) = \frac{B \alpha^2 d^2}{2\pi} \ln(\varrho / a_0) + 2 E_c(a_0), \tag{12}$$

where $\varrho = \sqrt{\xi^2 + \zeta^2}$ is the distance of the dislocations in scaled units and $a_0 \sim d$ is a short distance cutoff that signals the breakdown of continuum elasticity theory. The core energy $2E_c(a_0)$ plays the role of a chemical potential, i.e., the energy to create a defect pair at distance a_0 relative to a dislocation free system. Since there are many pairs, the effective interaction of a particular pair is a many-body problem. The pair carries a polarization cloud of bound pairs that screen the bare interaction. Following Kosterlitz and Thouless [11] this problem can be dealt with by the introduction of a scale-dependent dielectric constant $\epsilon(\varrho)$. The effective interaction then reads

$$U(\varrho) = \frac{B \alpha^2 d^2}{2\pi} \int_{a_0}^{\varrho} \frac{d\varrho'}{\epsilon(\varrho')\varrho'} + 2 E_c(a_0). \tag{13}$$

The probability $\Upsilon(\varrho)$ per unit area to find a pair at distance ϱ is then determined by the effective interaction $\Upsilon(\varrho) = a_0^{-4} \exp[-U(\varrho)/k_B T]$, or equivalently

$$\frac{d\Upsilon}{d\varrho} = -\frac{B \alpha^2 d^2}{2\pi k_B T \epsilon(\varrho) \varrho} \Upsilon(\varrho). \tag{14}$$

In terms of a scale-dependent stiffness $K(\varrho) = B\alpha^2 d^2 / [4\pi^2 k_B T \epsilon(\varrho)]$ and the dislocation fugacity y, where $y(\varrho)^2 = \varrho^4 \Upsilon(\varrho)$, this is the first of the celebrated Kosterlitz recursion relations

$$\frac{dy}{d \ln \varrho} = [2 - \pi K] y. \tag{15}$$

The renormalization of the stiffness is due to polarization of the dislocation pairs and is governed by the second recursion relation

$$\frac{dK^{-1}}{d \ln \varrho} = 4\pi^3 y^2. \tag{16}$$

The flow equations for the dielectric constant and the fugacity then reveal the existence of a low-temperature phase with a finite long-wavelength dielectric constant. Since $\epsilon(\varrho)$ depends intrinsically only on the flow parameter $l = \ln(\varrho/a_0)$ the first term in the effective potential is basically logarithmic and binds the pair only weakly.

In the presence of a strain s_x the pair is subjected to the Peach-Koehler force $E s_x d$ that separates the pair in z direction in addition to the attraction force corresponding to Eq. (13). For nonzero s_x the total potential exhibits a saddle point on the z axis and the pair can break up by escaping over the barrier as illustrated in Fig. 2. This creates a pair of free dislocations that contributes to the relaxation of the applied shear.

The motion of bound pairs is again given in terms of a Fokker-Planck equation [12]. Since climb motion is much

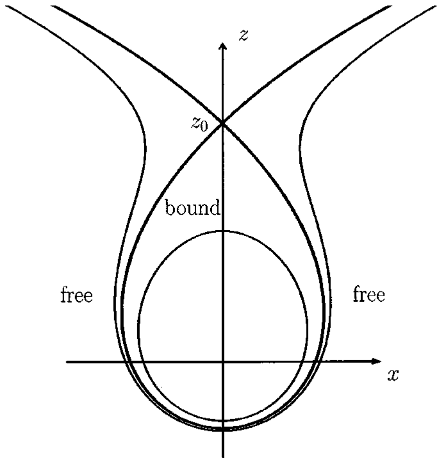

FIG. 2. Sketch of the effective potential for a dislocation pair in the presence of shear $k_BT\psi=U+Es_xzd$. The heavy line marks the boundary of bound- and free-dislocation pairs. Breaking of a bound pair occurs via escape over the saddle point. Since climb motion is negligible compared to glide the x coordinate is effectivley frozen.

slower than glide $\Gamma_x \ll \Gamma_z$ the associated currents are predominately unidirectional. With this simplification one derives

$$\partial_t \Upsilon = \partial_z \mathcal{J}_z, \quad \mathcal{J}_z = -2\Gamma_z \Upsilon[\partial_z U + Es_x d] - 2k_BT\Gamma_z \partial_z \Upsilon. \tag{17}$$

Since we are dealing with the *relative* motion of a pair, the diffusion constant is $2D_z$, where $D_z=\Gamma_z k_BT$ refers to diffusion of a single dislocation. The escape rate of this effectively one-dimensional problem can be obtained by standard methods. The saddle point $z_0=\zeta_0/\alpha$ of the total potential determined by the implicit relation $d+2\pi\epsilon(|\zeta_0|)\zeta_0 s_x\alpha=0$. For definiteness we discuss the case $s_x<0$, i.e., the saddle point lies on the positive z axis. The Fokker-Planck equation (17) implies that $\mathcal{J}_z$ is independent of the z coordinate. The solution can be obtained in terms of $\psi=[U+Es_xzd]/k_BT$ by writing

$$\mathcal{J}_z \int_z^\infty e^{\psi} dz' = -2k_BT\Gamma_z e^{\psi}\Upsilon|_z^\infty. \tag{18}$$

For $z=+\infty$ we expect that the dislocation probability density Υ vanishes rapidly, whereas for $|z|\ll z_0$ it assumes its equlibrium value. The right-hand side of Eq. (18) then simplifies to $2k_BT\Gamma_z a_0^{-4}$. The left-hand side is dominated by the saddle point and the integral can be extended to $-\infty$. Upon expanding near the saddle point and using the saddle-point equation, the Boltzmann weight can be approximated by

$$e^{\psi}=\frac{\zeta_0^4 e^{-2\pi K(\zeta_0)}}{a_0^4 y(\zeta_0)^2}\exp\left[\pi K(\zeta_0)\frac{x^2}{\alpha^2\zeta_0^2}-\pi K(\zeta_0)\frac{\alpha^2(z-z_0)^2}{\zeta_0^2}\right]. \tag{19}$$

Here we neglected terms involving the derivative of the dielectric constant, since these are small on large scales according to Eq. (16). The dislocation current is then found as $\mathcal{J}_z=R\sqrt{K(\zeta_0)}/(\alpha\zeta_0)\exp[-\pi K(\zeta_0)x^2/(\alpha^2\zeta_0^2)]$, where the production rate $R=\int_{-\infty}^{\infty}\mathcal{J}_z dx$ reads

$$R=2k_BT\Gamma_z\alpha^2 y(\zeta_0)^2\zeta_0^{-4}e^{2\pi K(\zeta_0)}. \tag{20}$$

For small strain s_x and not too close to the critical temperature, one finds $\zeta_0 \gg \xi_-$, where ξ_- is the correlation length implied by the recursion relations, Eqs. (15) and (16). Then $K(\zeta_0)$ can be safely replaced by its large distance limit $K(\varrho=\infty)=2/\pi[1+x(T)/4]$. Here $x(T)$ measures the distance to the critical point, and for temperatures close to T_c one has $x(T)\sim(T_c-T)^{1/2}$. Equation (15) then implies in this regime $y(\varrho)\sim\varrho^{-x(T)/2}$ and correspondingly the production rate exhibits a power-law dependence on the strain

$$R\sim|s_x|^{4+x(T)}. \tag{21}$$

The dynamics of the free dislocation density is governed by the rate equation

$$\partial_t n = R - rn^2. \tag{22}$$

The recombination process of two free dislocations of opposite charge is encoded in the rate constant r. Since local equlibrium is established much faster than dynamics of the broken and conserved variables, one can assume that the free dislocation density follows the rate adiabatically, i.e, $n \sim R^{1/2}\sim|s_x|^{2+x(T)/2}$.

A stationary, uniform external stress σ_{zx}^{ext} is balanced by a uniform strain Es_x, Eq. (5d). This results in a nonlinear dislocation flow $J_z\sim s_x^\gamma$, $\gamma=3+x(T)/2$, which gives rise to shear strain rates $\partial_x g_z=-J_z\rho d$. Upon collecting terms one derives the nonlinear constitutive equation

$$\partial_x g_z \sim (\sigma_{zx}^{ext})^\gamma, \tag{23a}$$

$$\gamma=3+x(T)/2. \tag{23b}$$

IV. CHANNEL FLOW

In this section we discuss the application of these ideas to flow of the stabilized smectic due to a pressure gradient along a channel. The film layers are oriented perpendicularly to the channel flow, i.e., the pressure gradient is along the z direction. Furthermore we assume that the channel is much wider than the correlation length ξ_- so that the hydrodynamic description is valid.

In the steady state all z derivatives vanish due to translational invariance along the channel, except for the pressure head $\pi'=-\partial_z p=\text{const}$. The equations of motion Eqs. (5) then reduce to

$$0=\partial_x g_z/\rho+\lambda_p E\partial_x^2 s_x + J_z d, \tag{24a}$$

$$0=\pi'+E\partial_x s_x+\nu\partial_x^2 g_z. \tag{24b}$$

Furthermore we impose the no-slip boundary condition $g_z(\pm a)=0$ and vanishing permeation current $\partial_x^2 s_x(\pm a)=0$, where $\pm a$ are the walls of the channel. Symmetry then implies $\partial_x g_z(0)=\partial_x^2 s_x(0)=s_x(0)=0$. The derivative $\partial_x g_z$ can now be eliminated from Eq. (24) yielding

$$-\pi' x/E=s_x-\delta^2\partial_x^2 s_x-\rho\nu J_z d/E, \tag{25}$$

where we introduced the permeation length $\delta=(\rho\nu\lambda_p)^{1/2}$. Since the dislocation current is nonlinear in the strain, the flow profile depends on the pressure head. The combination $\rho\nu J_z d/E$ can be written as $-s_x|As_x|^{\gamma-1}$, where A is a dimensionless constant. Rescaling of s_x reveals another characteristic length scale $L_\pi=E/(A\pi')$, apart from the channel width and the permeation length. There are several cases that can be discussed analytically.

For $L_\pi\gg a$, the pressure head is mostly balanced by elastic deformations of the smectic and the strain is given approximately by $s_x=-\pi' x/E$. The dislocation current is negligible compared to the elastic contributions, however, it gives rise to the fluid flow $\partial_x g_z=Es_x|As_x|^{\gamma-1}/\nu$. The flow profile then has the ineresting non-Newtonian form

$$g_z(x)=\frac{(\pi')^\gamma A^{\gamma-1}}{(\gamma+1)\nu E^{\gamma-1}}[a^{\gamma+1}-|x|^{\gamma+1}], \tag{26}$$

where γ is a continuously variable exponent in the range $0\leqslant\gamma=3+x(T)/2\leqslant 3$. In particular, the velocity is nonlinear in the pressure head contrary to conventional Poiseuille flow.

In the opposite case $L_\pi\ll a$, we assume that the permeation current can be neglected, except for a boundary layer of order δ. Then for $|x|\ll L_\pi$, one finds again $s_x=-\pi x/E$, whereas for $|x|\gg L_\pi$ one observes Poiseuille behavior $\partial_x g_z=-\pi' x/\nu$. The solution is self-consistent provided $\delta\ll a(a/L_\pi)^{(\gamma-1)/(2\gamma)}$.

V. CONCLUSION

We derived the hydrodynamic equations of motion for a smectic film stabilized by an external field. Below the Kosterlitz-Thouless transition dislocations are bound in pairs. However, a finite shear stress applied perpendicularly to the smectic layers nucleates dislocation pairs. The motion of these free dislocations results in a nonlinear viscous response. The smectic film exhibits shear-thinning giving rise to unusual flow profiles in channel flow.

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APPENDIX: ALIGNMENT OF THE NEMATIC DIRECTOR WITH THE DISPLACEMENT FIELD

The free energy Eq. (2) is the long-wavelength description of the smectic film. The nematic director θ is locked to the displacement via $\theta=-\text{const}\,\partial_x u$. This can be seen by taking a more microscopic approach by a free energy that includes the nematic director degrees of freedom explicitly, [10,11]

$$\tilde{\mathcal{F}}=\tfrac{1}{2}\int d^2r[B(\partial_z u)^2+D(\partial_x u+\theta)^2+K_0(\vec{\nabla}\theta)^2+\tilde{E}\theta^2]. \tag{A1}$$

Here D measures the free energy cost for nonaligned director to displacement, K_0 is the Franck constant and $\tilde{E}$ the coupling of the director to the external field. For an infinite system the contribution by the director field can be integrated out leading to an effective free energy in terms of the Fourier transformed displacement field

$$\mathcal{F}=\tfrac{1}{2}\int_q\left[Bq_z^2+Dq_x^2-\frac{D^2q_x^2}{\tilde{E}+Dq_x^2+K_0q^2}\right]|u(\vec{q})|^2. \tag{A2}$$

Expanding to lowest nontrivial order in powers of the wave vector $\vec{q}$ leads to the free energy Eq. (2) with $E=D\tilde{E}/(D+\tilde{E})$.

Near the boundary we use the more microscopic free energy Eq. (A1) and derive the corresponding Euler equations

$$D\partial_x u-K\nabla^2\theta+(D+\tilde{E})\theta=0, \tag{A3a}$$

$$B\partial_z^2 u+D\partial_x^2 u-D\partial_x\theta=0. \tag{A3b}$$

The first equation reveals the characteristic length scale $\lambda_0=\sqrt{K/(D+\tilde{E})}$. For variations of the displacement field with wave vectors $q\ll\lambda_0^{-1}$ the director field is given by $\theta=-\partial_x uD/(D+\tilde{E})$. Near the boundary where independent boundary conditions on the director and the displacement field can be imposed is a layer of order λ_0 where the previous relation does not hold. The second equation shows that the parameter entering the Euler equations corresponding to the effective free energy is $E=D\tilde{E}/(D+\tilde{E})$.

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